From the desk of Mogens Eliasen, for immediate release

November 30, 2004

Balancing the diet with logarithms…

When discussing food and balance in nutrition, it is important to understand that chemical processes in the body are controlled by the same fundamental chemical laws as the rest of the Universe.

One of the unpleasant consequences of this is that it gets impossible to understand how different chemicals impact each other, unless you comprehend that what matters for their energy and "value" for the body is not their concentration, but the *logarithm of their concentration*.

The chemical mass balance law

Most of the biochemical processes in the body are in fact equilibriums. This is simply a process that can run in both directions, subject to the current concentrations of the chemical compounds that are involved.

A simple way of describing this is by considering two chemicals, A and B that meet each other and react through some kind of exchange of energy or atoms that change both of them. Let's call the products that come out of this A′ and B′. This reaction is traditionally described symbolically by:

$$A + B \rightarrow A' + B'$$

But, the trick is that if we start with A′ and B′ and let *them* meet, some of them will convert back to A and B:

$$A' + B' \rightarrow A + B$$

We can shorten this to:

$$A + B \leftrightarrow A' + B'$$

indicating that the process can run either way, subject to what we have available for start.

The fundamental mass balance law, which is as basic as the law of gravity, now says that for such a reversible process, there is one and only one level of equilibrium (=balance). That level is reached when the two opposing reactions outweigh each other so the concentration of all four species, A, B, A′, and B′ are kept constant. This balance level is characterized by a dependency on the concentrations of the four involved chemicals that mathematically can be expressed as

{A}*{B} = constant* ({A'}*{B'})

where {X} refers to the chemical energy of X – which basically is a measure of the concentration of X.

We can also express the very same mathematical truth this way:

{A}/{A'} = constant * ({B'}/{B})

where we now emphasize that what matters here is the balance of the *ratios* of A and A' and of B and B'.

If A and B are very stable and thus not very reactive, then the constant is quite large – meaning that we do not get a lot of A' and B' produced. On the other hand, if the constant is small, then A and B will almost be used up as they generate a lot of A' and B'.

Let's put some numbers in, so we can see how this works. Let's assume that the constant is 1. When we mix equal amounts of A and B, say 10 of each (and none of A' and B'), then we can predict that a reaction must take place. The left-hand side of the equation is 10*10=100 – but the right hand side is 10*0 = 0! This imbalance can only be resolved by some of A and B reacting to generate A' and B'. You can probably quickly see that the only way we can satisfy the equation is by all four concentrations being equal to 5. (If we had been dealing with a constant that was different from 1, the result would have been different, as you can see. This equilibrium constant is a fundamental parameter that characterizes a chemical equilibrium, in a similar way as the melting point describes the temperature at which a solid material turns into liquid.)

We would get similar result by starting with 10 of each of A' and B', as you can see. The equilibrium is independent of how we start the processes!

But if we start out having this equilibrium of all species at a concentration of 5, what happens if we now add only one component, say A, in an amount of 10?

First, you can see from the math that the left-hand side now equals (5+10)*5 = 75 – which is much more than the 5*5 = 25 we have for the right-hand side. From this, we can predict with certainty that a reaction now must take place, so that the left-hand side is reduced and the right-hand side increases. This means that A must react with some of what remains of B to produce more A' and B'.

So, a fraction of A will react with what is left of B. It cannot *all* react with B, because there is only 5 left of B to deal with not only the original 5 of A but now also the additional 10! If you fiddle with the math, you will find that only 1.666 of A will react with 1.666 of B, so we end up having the following concentrations in the mix:

{A} = 13.333
{B} = 3.333

$\{A'\} = 6.666$
$\{B'\} = 6.666$

(You can check and see that both sides of the equation now come up as 44.444).

Now, here is the important conclusion we need to understand: we *doubled* the amount of A – and we increased A' from 5 to 6.666 – representing a 33% increase only! We did *not* double the effect by doubling the concentration of our input…. We only raised it 33%!

The fundamental mathematical reason behind this is that the mass balance is not determined by absolute amounts, but by **the *ratios* of concentrations.**

This is where the logarithm comes in. Logarithms allow us to deal with ratios and products as if they were subtractions and additions.

If we look at the mass balance equation once more, as we first wrote it, then we can use the logarithm on both sides of it and get this new representation of the same truth:

$$\log(\{A\}*\{B\}) = \log(1*\{A'\}*\{B'\})$$

Now, the fundamental feature of any logarithm is that the logarithm to a product of two numbers is equal to the sum of the logarithm of each of the two numbers in the product. So, this allows us to re-write the equation as

$$\log\{A\} + \log\{B\} = \log(1) + \log\{A'\} + \log\{B'\}$$

When you look at this, you can see the point in using logarithms when dealing with chemistry: when we add some of A and some of B, then what we get of A' and B' (and what will be left of A and B) is completely determined by how the *logarithms* of the concentrations add – not how the concentrations themselves add up. And it all comes from the fundamental fact that chemical reactions are determined by *mass balances or mass ratios*, not by the absolute amounts we consider.

The logarithm itself

There is a whole class of mathematical functions that are called "logarithms". They all share in common that when you operate them on a product of two numbers, the result will be equal to what you would get by operating the logarithm on each of the two numbers and then adding the two results. So, for our use in chemistry, it really doesn't matter which logarithm we use.

Each logarithm is completely and uniquely identified through the number that makes it assume the value 1. This number is called the base number for the logarithm. The common standard is to use the number 10 as base number. This means that we choose $\log(10) = 1$.

This is really all we need to know in order to calculate the logarithm of just about any number we want… We can easily calculate the logarithm of 100, because 100=10*10. So we get

log(100) = log(10*10) = log(10) + log(10) = 1 + 1 = 2.

Going the other way, we can see that log(1) must be 0, because

log(10) = log(10*1) = log(10) + log(1)

We cannot have more than one value for log(10) – so this implies that log(1) must be zero.

This will indicate to the smart reader that logarithms of numbers less than 1 should be negative. And is exactly the case. Let's take 0.1 as example:

log(0.1) = log(1/10)

Now, since log(1) = log(0.1*10) = log(0.1) + log(10) and log(1) = 0, then we get this true statement:

log(0.1) + log(10) = 0 or, rearranged a bit: log(0.1) = -log(10)

Instead of playing around with this yourself, I can just give you this overview of what the results will be:

X	1	2	3	5	10	20	50	100	1000	10,000	100,000
log(X)	**0**	**0.3**	**0.5**	**0.7**	**1**	**1.3**	**1.7**	**2**	**3**	**4**	**5**

X	0.5	0.3	0.2	0.1	0.05	0.02	0.01	0.001	0.0001	0.000,01
log(X)	**-0.3**	**-0.5**	**-0.7**	**-1**	**-1.3**	**-1.7**	**-2**	**-3**	**-4**	**-5**

(All the decimal numbers are rounded off – you really don't need all the digits – because there is no end to them – but this should give you a feel for how the logarithm works on numbers…)

Definition of pH

A very important application of this concept of using the logarithm of a chemical's concentration as a natural and relevant measure for its "strength" is given by the way we measure *acidity*. The definition of pH is "the negative logarithm of the acid concentration, measured in moles/litre", or

pH of X = -log{X}.

Let's take hydrochloric acid (which is the gas HCl dissolved in water) as an example. HCl has a molecular weight of 36.5, so 1 mole/litre means 36.5 grams/litre – which is a 0.365% solution (about a tenth of concentrated hydrochloric acid, which really is around 35-37% concentration).

The logarithm of 1 mole/litre is 0, so pH of a 0.365% solution of hydrochloric acid is 0. Since a concentrated 36.5% hydrochloric acid solution is ten times stronger, the logarithm of it will be 1 larger. But that means that the negative logarithm will be 1 less. So, the pH of concentrated hydrochloric acid will be -1.

Now, for every time we dilute this concentration by a factor of 10, we lower the logarithm by 1. But that means *increasing* the *negative* logarithm by 1!

So, the more we dilute this acid, the higher the pH goes. The pH value will increase by one for every time we dilute the acid to a tenth of what it was.

However, when we come close to the acidity that exists naturally in pure water, this dilution principle won't continue. Pure water will naturally have a pH of about 7- that's the balance point between acid (H_3O^+) and alkaline (OH^-) in pure water. This balance can be expressed as

$$H_2O + H_2O \leftrightarrow H_3O^+ \text{ (acid)} + OH^-$$

Doesn't that look familiar? Well, it should tell you that, by taking acid away from this balance, we can never get past the point of it being regenerated by water reacting with itself, since there is more than a million times as much water as acid! This means that, when we have diluted our acid almost into eternity, we can never get pH any higher than 7. To get past that, we need to add alkaline….

As you can see, the pH definition follows exactly the concept of using the logarithm of the concentration of a chemical, here an acid. The pH scale is a nice linear scale – representing a huge span of possible concentrations from pure concentrated matter down into millionths of a percent. Not only would it be extremely impractical to work directly with those numbers (too many zeroes to count!) – but they would actually also give us a false idea of their relative importance. The logarithmic representation is simply a nice solution to that, once you stop being scared of the math…

I hope this little overview has helped on that fear.

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